Developing Year 5 Students' Understanding of Density: Implications for Mathematics Teaching

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Promoting the concept of density is regarded typically as the domain of science teachers. However, density as the relationship between an object's mass and volume highlights the importance of foundational measurement concepts located in the mathematics curriculum. In this study, an integrated mathematics and science unit on density was developed for students in the middle years of schooling (Years 4-9) by the research team. Implementation of this unit in Year 5 classrooms is reported here. The lesson sequence appeared to support students' capacity to use the language of density in meaningful contexts. Implications for teaching are drawn from the data.

Density as a Special Ratio

In research about proportional reasoning, the difference between extensive and intensive quantities has been highlighted. Extensive quantities, according to Singer, Kohn and Resnick (1997) quantify how much, or how many. Mass, volume and heat are examples of extensive quantities (Fassoulopoulos & Koumaras, 2003) and are additive in nature (a mass of 17 kg and 22 kg will combine to a total mass of 39 kg). Intensive quantities are two extensive quantities that are combined to form a special ratio and they do not behave additively (Singer, Kohn and Resnick, 1997). Nunes, Desli and Bell (2003) have drawn educators' attention to the fact that early schooling provides students with experiences to build understanding of extensive quantities but stated that little time is devoted to working with intensive quantities.

Density is the ratio of an object's mass to volume. Two extensive quantities, mass and volume, combine to form the intensive quantity density. The formal definition of density is mass per unit volume and is represented by the formula:

$$\rho = \frac{m}{V} \text{ (units of g/mL or kg/m}^3\text{)}$$

Clearly, an understanding of mass and volume underpins an understanding of density. In science, mass is one of the fundamental concepts and corresponds to the amount of matter in an object. The mass of an object is constant in everyday life provided that no matter is added or lost. This is the basis of the Law of Conservation of Mass which indicates that whenever there is a change, the total mass before the change is the same as the total mass after the change. The mass of an object is always the sum of the masses of its component parts. In mathematics, mass is a core concept within the measurement strand. Early activities focus on comparing and ordering objects with different masses and quantifying mass with non-standard units before standard units are introduced. Precision in measurement is explored using various measuring devices. Experiences in measuring mass can actually be a stumbling block for students because of the way it is measured indirectly. In science terms, what is measured is a force (the gravitational attraction of the object to

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the Earth) which is proportional to the mass. The mass can be calculated from the force (the weight) by calibrating the scales appropriately in the units of mass. Often the terms weight and mass are used interchangeably, causing confusion for students when they start to learn about forces.

In both science and mathematics, volume is the three-dimensional concept of the amount of space occupied by an object. The volume of any object can be measured by displacement, and the volume of regular objects can be calculated from measured dimensions. Density, the ratio of mass to volume, is a derived property. Part of the utility of density as a property arises because it is an intensive property (which is independent of the amount of the substance), unlike mass and volume which are extensive properties. The value of this idea is seen when we say that the density of water is one (g/mL) whereas to say that the mass of water is 1 kg has little meaning.

Proportional Reasoning for Density

It is generally accepted in the literature that promoting students' understanding of density has a long and problematic history (Hitt, 2005; Libarkin, Crockett & Sadler, 2003; Maclin, Grosslight & Davis, 1997). This is acknowledged partly because understanding density requires a measure of proportional reasoning (e.g., Singer, Kohn & Resnick, 1997). According to Behr, Harel, Post & Lesh (1992), proportional reasoning is understanding the multiplicative relationship inherent in situations of comparison. In the case of density, as highlighted by Nunes, Desli and Bell (2003), density as an intensive quantity involves "one extensive quantity that is directly proportional to the intensive quantity and a second one that is inversely proportional to it" (p. 662). Further, they state:

> In the case of density: for the same volume, the greater the mass, the greater the density (direct relation) whereas for the same mass, the greater the volume, the smaller the density (inverse relation). Density is defined as mass divided by volume. (Ibid)

Promoting students' understanding of density as an intensive quantity can be supported through considering why particular objects sink or float. The tendency of an object to sink is directly proportional to its mass and inversely proportional to its volume. The relationship is captured in comparing an object's mass and volume (which can be expressed as a number through division: m/V) which then enables students to think about sinking or floating rather than the underlying concepts of mass and volume. However, it is only useful for solving everyday problems if the relationship between mass and volume is well understood. For example, in order to float for a few minutes on the surface of a swimming pool, a deep breath will inflate the lungs and increase volume without increasing mass.

Implications for the Study

Beyond development of density as a special ratio and an intensive quantity, density is also a property that is more abstract than mass or volume as it is about the nature of the physical material of the object and how 'tightly packed' the material is (Kohn, 1993). Density in relation to properties of materials draws attention to a further dimension of this concept and the interconnectedness of mathematics and science. It raises questions relating to teaching mathematics and science in the middle years of schooling. Can mathematics and science teachers communicate better to support students in developing understanding of density? What might an integrated unit on density look like? What prerequisite mathematics knowledge is required to support understanding of density? In this report, the last two questions were the research questions that guided this study. The first question is rhetorical but can be partially addressed through consideration of the implications of this research.

The Study

The research reported here is part of a larger study on promoting students' proportional reasoning in mathematics and science. The project involves six schools across each of the educational sectors (State, Catholic and Independent) and includes single-sex schools, low socio-economic status schools, high socio-economic status schools. The participating teachers include primary teachers, secondary mathematics-only teachers, secondary science-only teachers and secondary mathematics and science teachers. Collectively, the year levels taught, span Years 4-9.

The aim of the integrated unit of work was to further promote connected teaching across mathematics and science. The unit was designed so that it would favour equally mathematics and science core learning outcomes for the middle years. In Queensland, the curriculum is derived from Essential Learnings that "identify what should be taught and what is important for students to have opportunities to know and be able to do. They [Essential Learnings] describe the *ways of working,* and *knowledge and understanding* that students need for ongoing learning, social and personal competence, and participation in a democratic society (QSA, 2007, p. 2). Analysis of the Essential Learnings showed that this unit addressed mathematics knowledge and understanding of number, measurement, space and chance and data; and science knowledge and understanding of natural and processed materials and the nature of forces. Ways of working in both mathematics and science included investigating, thinking and reasoning, communicating and reflecting.

In its original form, the unit comprised 8 lessons of approximately 35-40 minutes duration and included a pretest/posttest for diagnostic and comparative purposes, and a culminating investigative assessment task. Extension and alternative pathways were also suggested. The intention was for teachers to implement the unit with their own classes and to feedback to the project team on its value as well as to suggest modifications and adjustments for particular grade levels. Project teachers who taught primary grades were initially concerned that the content and structure of the unit would be too daunting for their students, particularly as, in these grades, volume is restricted typically to liquid volume via displacement rather than cubic measures. They also stated that the concept of density would be too advanced for their students. Upon reassurances that we thought this was not the case, Year 5 teachers at one school asked if a member of the research team could implement the unit with their students so that they could observe.

Design

A design-based research approach was adopted with its continuous cycle of design, enactment, analysis, redesign (Design-Based Research Collective, 2003). The design of the unit drew upon Donovan and Bransford's (2005) three principles of learning that emphasise considering students' prior knowledge and their preconceptions derived from experience outside the classroom; facts, knowledge and conceptual frameworks required for competence in a domain; and supporting metacognitive processes. The lessons were designed to provide opportunities for students to work actively with materials and to discuss ideas to build rich conceptual knowledge and understanding. Using a constructivist framework, the unit was designed to provide engaging and intellectually challenging learning environments and activities so that students would actively construct deep understanding of the concepts and issues being explored.

All lessons were videotaped, and three roving video cameras enabled specific dialogue of students to be recorded during group work. Video footage was viewed by the research team to analyse the extent to which teacher actions and planned activities promoted and prompted students to demonstrate growing awareness of density, volume, and mass, and the extent to which the lessons specifically aligned Donovan and Bransford's (2005) principles of learning. The culminating task resulted in students developing a PowerPoint presentation that was taken as a measure of their capacity to describe density in context.

Results

The diagnostic pretest served to provide a quick (10 minute) survey of students' current knowledge of density, mass, volume. Most students could describe mass but predominantly stated that mass meant "weight". To describe volume, some students drew diagrams of containers, but most explanations related to capacity ("space inside", "the contents of something", "how much it contains"). Density explanations were sparser than volume explanations and included "pressure on something", "solids and liquids", "more weight", "when something is solid, no air". The posttest revealed greater student confidence in describing all three terms. Students described mass as heaviness, volume was described in terms of liquid displacement and cubes, and density included words volume and mass, with some students writing the formula.

The introductory activity commenced with the teacher asking students to brainstorm a list of objects that sink or float in water, which were written on the board. In all three classes, 'a rock' was the first object that students nominated as an object that sinks in water. Through teacher prompting, students were challenged to state what makes an object sink or float in water. Predominantly, mass was the first factor nominated. Through being provided with counter-examples by the teacher (a heavy piece of wood, a pumice stone, a ship), the students said it might have something to do with the type of material, its colour, the amount of air that can get into it, the 'circulating fluids' inside an object. One student suggested density, but when asked "Do you know how that works?", replied, "Nope!" We suspect this word arose due to the pretest on which density featured quite extensively. Students were then given pairs of objects of comparable size (apple and potato; pebble and wooden block) as these objects were large and light, large and heavy, small and heavy, small and light respectively. Students had to predict which objects would sink or float and state why. They then placed each object in a tub of water, recorded the results and wrote a reason for the outcome. In the concluding discussion, the teacher drew students' attention to volume and mass of the objects, gently inserting these words when students said things such as "it's heavier than the others", "it has something to do with size".

For the next lessons, a set of purpose built, opaque "Density Jars" featured (a set of 6 identical jars of the same volume but different mass and a set of 5 jars with the same mass, but different volume). The jars are similar in shape for ease of measuring volume by calculation (area of base x height). When first introduced to the jars, students were asked to order the jars according to mass, which they determined first by heft and then by measuring with scales. Through this activity, students realized that the density jars of the same size all differed in their mass, but the jars of differing size all had the same mass. Errors in measurement were discussed as a class data table was constructed.

The teacher then drew students' attention to volume, asking students to define volume and nominate the units in which volume was measured. Some students described cubic ‘animals’ that they had constructed from blocks prior to this unit. They mentioned that they had counted the blocks and discussed volume. With some prompting and the teacher mentioning displacement, the students began to describe prior experiences in dropping objects in water and measuring the amount of water displaced. The teacher reminded students that volume could be measured as a liquid volume (mL) or a cubic volume (cm^3) and showed students a MAB unit block to assist conceptualization of cubic measures. The teacher then showed students an MAB 1000 block and drawing students’ attention to the fact that it comprised 10x10x10 unit cubes, asked students to state its volume. After ascertaining that all students knew its volume was 1000 cm^3, the teacher then displayed a 10x10x10 cubic centimeter measuring cube that completely enveloped the MAB 1000 block. A one litre bottle of water was then poured into the measuring cube and students could see that 1000 L was equivalent to 1000 cubic centimeters. The teacher reinforced the relationship that 1cm^3 is equivalent to 1mL. The teacher then held up a can of soft drink and asked students to estimate its volume. The can was then placed in water and liquid volume ascertained and converted to cubic volume. This was repeated with two other objects. The students readily engaged with this activity, enjoying the challenge of checking their estimations. Volume as the amount of space an object takes up and that this could be described as a liquid or cubic measure was continually emphasised by the teacher.

For the next part of the unit sequence, the plan was for students to find the volume of each of the density jars, but to do this via displacement would enable them to determine which objects sink or float, and this would negate the need to compare the relationship between an object’s mass and volume. As the students had yet to explore the formula for volume (area of base x height), finding cubic volume was not a possibility for these Year 5 students. The teacher then drew students’ attention to each of the density jars, asking students to make a prediction of the volume of each one in cubic and liquid volume. The teacher then provided students with the actual volume and all students recorded this in their data table, which was also recorded on the board. The list was organised so that students could see that the first set of jars had constant volume but gradually increasing mass; and the second set of jars had constant mass but increasing volume. Students were then required to make predictions of which jars would sink and which jars would float and then they tested their predictions. Before all the jars had been tested, the teacher asked students to identify a tested jar that sank and a jar that floated. The students confirmed for the teacher that one of the jars had a mass of 500 g and a volume of 600 cm^3 and that it had floated, but the jar with mass of 500 g and a volume of 400 cm^3 had sank. With urging, the students started to make predictions on the basis of whether the mass was greater than the volume or the volume was greater than the mass. The students conscientiously placed each of the jars in the water one at a time, with all group members stating whether they thought it would sink or float after checking their measurements of volume and mass. A feeling of high excitement was evident as students congratulated themselves on predicting correctly: “See, I knew that I would be right!” At the end of the activity, the teacher asked students to describe what things needed to be considered when predicting whether an object would sink or float. The teacher formally introduced the term density at this point, emphasizing that density was determined by comparing the mass of an object to its volume.

In the next lesson, students were provided with a hollow plastic cylindrical container with a lid. The container had a hole pierced in its sides and a metal weight on its end. When first put into water, the vessel would float, but would gradually fill with water and sink to the bottom. The students were required to take a series of photographs of this vessel (named the Titanic) at various stages on its voyage to the bottom of the ‘ocean’, and to use the volume, mass and density in a PowerPoint that told the story of why their vessel sank in water. The students were very adept at taking and loading digital photos onto computers and making PowerPoint shows. This task was accomplished in approximately one hour. All groups' presentations showed appropriate use of the terms mass, volume and density to match their series of 4-6 photographs.

Discussion

The unit was designed to target the following core teaching points/activities: (i) introductory exploration "What sinks/floats in water" (ii) measuring mass (iii) linking 1 cm^3 to 1 mL (iii) estimating volume by comparing to MAB 1000 block (iv) finding volume (displacement and/or measuring and calculation) (v) comparing mass and volume of particular objects and prediction of sinking or floating (vi) relationship of density to value of 1. In the compressed time frame, students were provided with activities to enable experience with these key teaching points. At the end of the sequence, students were competently using the terms mass, volume, and density and reconsidering their initial ideas on why particular objects sink or float in water. Due to the restrictions on time by the school, more directed teaching occurred than was considered ideal. The first example was the quick demonstration of pouring 1 litre of water into a 1000 cm^3 container in order to introduce equivalence of measures between liquid and cubic volume. Although pedagogically sound, it would have been good for students to experiment more with this notion by performing their own measuring trials. Also, the actual real world conditions which affect this relationship were not discussed in any way (temperature of water). The second examples was when it became apparent that students' conceptualisation of volume was not well established. Students were not making direct connections to their recent experiences of cubic animal building activities and previous volume by displacement activities. Providing the MAB 1000 block as a visual referent was for the purpose of assisting students to estimate volume. Whether visualising in cubes or in liquid millilitres assists more accurate estimation was not something that was explored here. A more student-centred experience could have been included to help students spend more time with this notion, but again, time did not permit this.

Other major ideas were not fully elaborated due to time restrictions. Units of measure for volume and mass were not fully explored. The students appeared to be operating at a numerical level only when they measured the mass of each of the density jars, with many students' data tables showing entries that omitted the units. When providing students with the volumes of each of the density jars, the teacher included the liquid volume units (mL) after stating first the cubic volume units (cm^3). The teacher reinforced this label and required the students to insert this in their data table. However, when the students then came to compare the mass and volume of each of the jars, the units were ignored. By ignoring the units and comparing the mass and the volume as two numbers, the students could determine whether an object would sink or float. The term density was provided, but there was no time to emphasise the importance of checking units before determining ratio (g/mL; kg/L). The students clearly used an additive comparison when comparing mass to volume: if mass is less than volume, the object will float; "if the mass is greater than the volume, the object will sink." When the teacher introduced the word density, the formula for finding density (m/V) was presented and the students calculated this number using calculators and entered this on the data table. Students readily determined that an object with a density less than 1 (one) would float. Introducing the ratio formula may be considered "rushed' for students at this stage, but it readily provided an avenue for further analysis of the data table and for making generalisations about density. Precision in measurement was also not explored, and this would have been an issue if the jars had not been carefully designed to ensure that the numerical value of the mass was quite different to its volume because some objects.

Although the identified aspects of the lesson sequence show points in time where teaching was more directed than ideal, it also shows the strong foundational structure that was laid for students to work sensibly with the concepts of density as a relationship between an object's mass and volume, and this was seen in their powerpoint presentations. The following quote by Behr, Harel, Post and Lesh (1992) suggests that explicit teaching can assist connected knowledge growth, which was our intention here:

> It does not follow [however], that students cannot or should not receive mathematical knowledge from their teachers, that mathematics instruction should not be organised to facilitate the teacher's clear presentation of knowledge, that the structure of mathematics should not provide a basis for sequencing topics of instruction, or that mathematical skills cannot be integrated and taught along with student understanding and problem solving. (p. 326)

By further analysing the structure of the teaching sequence, aspects of Donovan & Bransford's (2005) principles of learning were evident. The introductory activity enabled students to share their general perception that heavy things sink and light things float. They were challenged to consider and reconsider their position through teacher questioning. They were also continually asked to explain their thinking and reasoning for any answers they provided in class discussions. The provision of calculation of density through formula also provided meaningful skill practice in context, and students' end products (Titanic) indicated conceptual frameworks for density concepts as students' demonstrated their capacity to use the language of density meaningfully.

Conclusions and Implications

As a result of this unit being taught in their classes, the Year 5 teachers in this project realised their students' capacity to access this domain and operate meaningfully in it. Their initial objection had been in relation to the fact that they could not find density listed anywhere within the science syllabus documents for this year level. In fact, it is not specifically listed anywhere in the Essential Learnings for Queensland but would be located in the strand of Natural and Processed Materials that is concerned with properties of matter. Notwithstanding this argument, the unit caused the Year 5 teachers to reflect on their own beliefs about their students' potential for complex ideas. It also highlighted the importance of teaching mass and volume. With richer and more extensive experiences with volume, the students would have come to this unit with a more ready conceptualisation of volume as the amount of space that an object takes up. It is foundational to density conceptualisation from which further knowledge, particularly in science, can grow.

Although the obvious goal of this unit was for students to develop an understanding of density, the mathematical foundations required were highlighted. Apart from mass and volume, number knowledge of fractions and decimals would support more ready analysis of mass compared to volume as a division, which could then be related to 1 (the density of water). We believe that few students in this study would have had prior experience estimating or dividing bad and ugly numbers such as 550 by 650 (or in fractional terms 550/650) and knowing how close to 1 the resulting number would be. Students' understanding of units of measure was also not well-established, suggesting greater need for units to be the focus of measurement activities in the mathematics classroom.

The topic of density has been identified as a way of promoting students' proportional reasoning (e.g., Richardson, Matthews, & Thompson, 2008). Indeed, research reported by Maclin, Grosslight & Davis (1997), indicated that through an extended unit of density, students performed well on proportion tasks relating to sweetness. Density is a special ratio and an intensive quantity and is more than knowing the multiplicative comparison between two quantities. We cannot say the extent to which students have developed this level of proportional reasoning in relation to density as a result of this very compressed teaching sequence, but there is evidence that students could operate competently in this domain. Nunes, Desli and Bell (2003) have called for mathematics teachers to extend students', and particularly young students', experiences with extensive quantities to include exploration of intensive quantities. Our study here has shown that carefully planned lessons can support students' exploration of density as an intensive quantity, and that lesson design can compensate for short-comings in students' experiences that may prevent access to the domain. Of more importance however, is the sort of prior mathematical experiences we have identified that enhance a science unit on density.

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